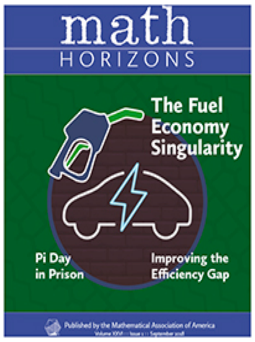

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John Bukowski and Sanny de Zoete

Figure 1. Balthasar van der Pol.

The city of Leiden, the Netherlands, is home to a well-known university and several excellent museums. Among them is Museum Boerhaave, the Dutch National Museum for the History of Science and Medicine. Recently John visited its library to explore the archive of the 20th century Dutch mathematician-physicist-engineer Balthasar van der Pol (figure 1).

The items in the archive demonstrate Van der Pol's varied interests. One finds the expected correspondence (with mathematician and historian Dirk Struik and Nobel Prize-winning physicist Hendrik Lorentz, among others), mathematical manuscripts, and lecture notes. There are also several newspaper clippings of chess problems, some solved by him and some submitted by him. The archive also contains music written by Van der Pol, who was a violinist. There are manuscripts and published songs, and pieces written for violin and piano. The most surprising—and fun—part of the collection, however, is the set of four *tafelkleedjes*. Tafelkleedjes? Tablecloths? What kind of a mathematician has tablecloths in his archive?

Van der Pol

Balthasar van der Pol was born in Utrecht, the Netherlands, in 1889. He studied physics and mathematics at Utrecht University, graduating in 1916. He then earned his doctoral degree at Utrecht in 1920 in the field of electromagnetism and radio waves.

Students in a differential equations class likely know of Van der Pol from the second-order nonlinear equation that bears his name:

$$y'' - \varepsilon(1 - y^2)y' + y = 0,\ \varepsilon > 0.$$

The Van der Pol equation describes so-called relaxation oscillations in a particular kind of electric circuit, as he discovered in the 1920s.

Van der Pol worked from 1922 to 1949 at the Philips Research Laboratory in Eindhoven, where he became Director of Scientific Research. He was known throughout the world as a leader in radio engineering. While at Philips, he also served as a professor at the Technical University of Delft, and he was later a visitor at the University of California, Berkeley, and Cornell University. He died in the small Dutch town of Wassenaar in 1959.

The Tablecloths

Inspecting the folded tablecloths—which are still in their original plastic wrappers—one sees a pleasing pattern of small colored squares with four-fold symmetry around a central point (see figure 2). Turning the package over reveals that the tablecloths show the Gaussian primes in the complex plane. There is a detailed explanation of the Gaussian primes and the pattern, with the signature "BALTH. VAN DER POL, GENEVA, March, 1954" at the bottom.

Van der Pol begins with an introduction to the Gaussian complex integers.

> The Gaussian complex integers are defined by $m + in$, where both m and n run independently through all positive and negative ordinary integers (zero included) and $i = \sqrt{-1}$.

He then shows that the product of two complex integers is also a complex integer, before beginning his discussion of prime numbers.

> As is well known, in the field of (ordinary) rational integers, not every integer can be

decomposed into smaller factors; the ordinary integers which cannot be decomposed are the ordinary primes. The same is true in the field of the complex integers. Thus the complex integers which cannot be written as the product of two other complex integers (ignoring the factors 1, -1, i, and $-i$) are called the complex, or Gaussian, primes. The latter are marked as little squares on the tablecloth.

Van der Pol goes on to explain that some of the ordinary primes we know are also complex primes, but some of them are not.

It can be shown that ordinary primes such as 3, 7, 11 which are of the form $4k-1$ (k integer), remain primes in the Gaussian field and they are therefore marked as such on the real and complex axes. However, the ordinary primes of the form $4k+1$ such as 5, 13, 17 are no longer primes in the Gaussian field, because they can be decomposed as follows

$$5 = (1 + 2i)(1 - 2i),$$
$$13 = (3 + 2i)(3 - 2i),$$
$$17 = (4 + i)(4 - i),$$

and therefore the ordinary primes such as 5, 13, or their associates, $5i$, $13i$, $-5i$, $-13i$, -5, -13, are not marked on the axes.

Finally, Van der Pol writes very briefly about the complex integers not on the axes—the vast majority of the diagram, and the part that makes it a beautiful pattern: "Further, numbers like $3+2i, 5+4i$ are also complex primes and are marked as such on the diagram." We can see these two complex primes marked in orange in figure 3.

Van der Pol concludes his description of the tablecloth by stating an interesting fact about these types of primes. To understand Van der Pol's remark we need to know that the modulus of the complex number $m+in$ is $\sqrt{m^2+n^2}$. Van der Pol states that the square of the modulus of these primes "is an ordinary prime of the form $4k+1$," giving the examples

$$3^2 + 2^2 = 13,$$
$$5^2 + 4^2 = 41,$$

related to the numbers $3+2i$ and $5+4i$ mentioned previously. We note that this property does not hold for $1+i$, $1-i$, $-1+i$, and $-1-i$, since the square of the modulus is 2, which is not of the form $4k+1$.

History of the Prime Cloths

We begin the story of the tablecloths in September 1954 at the International Congress of Mathematicians (ICM) in Amsterdam. At this meeting, Jaap van Dissel, managing director of the linen manufacturer E. J. F. van Dissel from Eindhoven, sold these prime number cloths of Van der Pol. The square cloths measuring 70 × 70 cm came in four colors: red, green, blue, and yellow. The conference attendees liked the cloths very much, and they sold quite well. This is certainly the source of the tablecloths at the

Figure 2. Van der Pol's tablecloth.

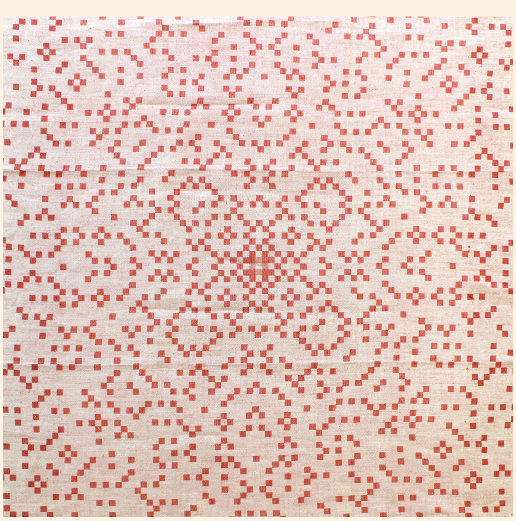
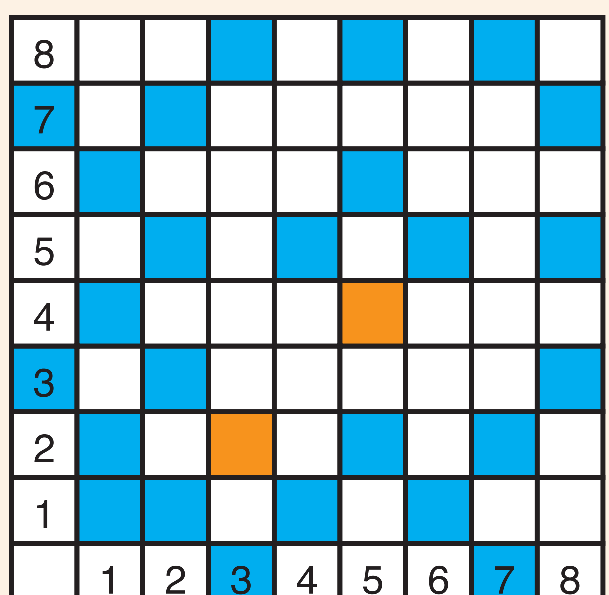


Figure 3. The complex plane with the Gaussian primes colored. The primes 3+2*i* and 5+4*i* are orange.



Museum Boerhaave, but is it the beginning of the cloths' history?

Also in the Museum Boerhaave archive is a letter from Van der Pol to the number theorist Johan van der Corput, with which Van der Pol sent one of the cloths. In the letter Van der Pol explains the Gaussian primes and tells Van der Corput that Van Dissel has woven some of the cloths with the intention to market them in the near future. The letter was dated April 26, 1946, nine years before the ICM.

But the story goes back even further than that. Last year Van der Pol's grandson René Roessingh sent us the published notes of a lecture by Van der Pol from November 1945 in The Hague for the learned society Maatschappij Diligentia (B. Van der Pol, *Merkwaardige eigenschappen van geheele getallen*. 's Gravenhage, Netherlands: W. P. van Stockum and Zoon, 1946). After making jokes about Dutch expressions with numbers, Van der Pol discussed the importance of the subject of number theory. He then introduced the ideas of factors and prime numbers. After a brief mention of perfect numbers, he returned to the topic of prime numbers, discussing the sieve of Eratosthenes and the distribution of primes.

Van der Pol then introduced the Gaussian integers and Gaussian primes in a manner similar to that described previously. At this point in the lecture, he showed the audience one of his prime number cloths, saying that it is "not only mathematically interesting, but also striking because of the beauty of the pattern." He noted that one could see in it "new combinations and designs forever." It is surprising that in November 1945 Van der Pol showed this cloth because of the scarcity of linen yarns in the Netherlands, just after the end of the Second World War.

Interest in the Prime Cloths

In 1955 the topologist Hans Freudenthal wrote a long article in the Dutch newspaper *De Groene Amsterdammer* entitled "Prime numbers and textile." ("Priemgetallen en textiel," [March 5, 1955]. Translated by Diane Webb.) After a discussion of prime numbers and an explanation of the Van der Pol cloths, he ends by praising the beauty of the cloths:

> Take a good look! It's a beautiful pattern. A pattern that can be continued into infinity, with new variations and surprises at every turn, with a capriciousness that defies all human imagination and yet follows a fixed, inexorable rule. Plato would have exclaimed that it was the pattern followed by the Creator in modeling the world. Let us put it more humbly: A pattern that is older than the world and will not perish with the world.

At about this time, one of the world's most famous scientists enjoyed the prime number cloths of Van der Pol. In 1954, Jaap van Dissel sent one of the cloths to Julien Elfenbein for publication in the magazine *Linens & Domestics* because it was "unique and never done before" (Kestenbaum & Co., Auction 58 [2013], *bit.ly/VanDerPolAuction*). Elfenbein asked for a second cloth and sent it as gift to Albert Einstein. Elfenbein wrote, "Even with the explanation included I am still in the dark as to what it means. I am sure you will find it enlightening." Acknowledging receipt of the special cloth, Einstein thanked Elfenbein in a letter and added, "May I ask you to send my thanks and appreciation also to Mr. Van Dissel."

A few years later, *Fortune* magazine dedicated a full page to the cloths, showing a large diagram of the pattern and giving an explanation (G. A. W. Boehm, "The new mathematics," [June 1958]: 140–158). Part of the diagram is reproduced in figure 4—notice the numbers on the colored squares. The first explanation given is different from Van der Pol's as it does not use complex numbers. The magazine begins, "When divided by 4, all primes, except for 2, leave a remainder of either 1 (the $4n+1$ primes) or a remainder of 3 (the $4n+3$ primes)." Note that these $4n+3$ primes

Figure 4. A partial reproduction of the diagram from *Fortune*.

	113		89		73			8			73		89		113	
113						53		7		53						113
			61				37	6	37				61			
89		61		41		29		5		29		41		61		89
			41				17	4	17				41			
73						13		3		13						73
	53		29		13		5	2	5		13		29		53	
		37		17		5	2	1	2	5		17		37		
8	7	6	5	4	3	2	1		1	2	3	4	5	6	7	8
		37		17		5	2	1	2	5		17		37		
	53		29				5	2	5		13		29		53	
73						13		3		13						73
			41				17	4	17				41			
89		61		41		29		5		29		41		61		89
			61				37	6	37				61			
113						53		7		53						113
	113		89		73			8			73		89		113	

are the $4k-1$ primes mentioned by Van der Pol shown previously.

The magazine then explains, "All the $4n+1$ primes can be expressed as the sum of the squares of two numbers [related to Van der Pol's explanation] . . . but none of the $4n+3$ primes is equal to the sum of the squares of two numbers." They then explain that "the prime 89, which is equal to 8^2+5^2, is plotted at eight points: (8,5), (5,8), (8,−5), (−5,8), (−8,5), (5,−8), (−8,−5), and (−5,−8)." Notice in figure 4 that the number 89 appears in the squares corresponding to each of these eight locations.

Only after this explanation is complete does the magazine go on to introduce imaginary numbers and to give a quick version of Van der Pol's original explanation. The end of the description gives the address of the manufacturer in Holland, offering the cloths at only $2 each.

Sanny's History with the Prime Cloths

But that was not what Sanny paid in 1996 for her first cloth. "800! 900! Nobody more than 900 guilders?" said the auctioneer, "Nobody more?" Tap, and with a strike of the hammer she was the new owner of a dizzyingly expensive cloth (about $1,000) that had something to do with math—but she had no idea what.

A Dutch collector of linens once told Sanny that there had been a cloth made in Holland with irregular small squares in it, and that it was quite special. So when Sanny saw such a cloth at an auction she decided to try to buy it—for 30 or 40 guilders, she thought. But she was totally wrong, as somebody else also knew this was a very special cloth. After she bought it, she researched the history of the cloth. She spoke with Van der Pol's second wife and Jaap van Dissel's son Wim.

At that time Sanny was a collector of antique linens and damask household textiles, but in 1997 she started to produce fine-quality linen damask tablecloths and household textiles designed by Dutch artists. (See *www.sannydezoete.nl/en/damask/primes.*)

In 2004 when Sanny opened a shop in Delft, home of the famous Dutch porcelain as well as the Technical University where Van der Pol once taught, it seemed to be a good idea to bring the prime cloth into production again. With permission of the Van der Pol and Van Dissel families, she sat for many evenings drawing a quarter of the design on graph paper and making sure each particular square was in the right position.

She wanted to make the new cloths approximately the same size as the original one with a good quality of linen and cotton yarns. However, due to the size of modern-day looms, Sanny's cloths are slightly smaller, at 67 × 67 cm (see figure 5). There have been no linen manufacturers in Holland since Wim van Dissel closed the nearly 100-year-old linen factory in 1970. So the new cloths were woven in Belgium as they still are today.

Figure 5. Sanny's reproduction of Van der Pol's tablecloth.

We are glad to introduce the beautiful prime number cloths of Van der Pol to a new audience, so that people can once again enjoy looking at the pattern and puzzling over its meaning.

John Bukowski *is professor and chair of the department of mathematics at Juniata College. He is interested in the history of mathematics, specifically the work of Christiaan Huygens. He fortuitously came across the Van der Pol tablecloths during a recent visit to Leiden.*

Sanny de Zoete *is an independent art historian with expertise in the history of linen and damask production in Holland. She collects linen damask and is producer of Dutch Design Damask. She makes exhibitions and writes about this subject. She lives in Delft, the Netherlands.*

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